TecnoCampus
Escola Superior
de Ciències Socials i de l'Empresa

Centre adscrit a la
Universitat Pompeu Fabra Barcelona

EOQ: Optimizing price and order quantity for growing items with imperfect quality and carbon restrictions

Nom de l'estudiant: Clàudia Faraudo Pijuan

Nom de la tutora: María de Lourdes Eguren Martí, PhD

Data: 20/06/2021

MEMÒRIA DEL TREBALL FINAL DE GRAU

Curs: 4t

Estudis: Logística i negocis marítims

Resum

Aquest treball de final de grau busca crear un model de gestió d'inventari basat en el format EOQ (ordre econòmica de quantitat) que optimitza el cicle de l'inventari així com el preu final dels productes. Està adaptat a les necessitats de la indústria ramadera i agrícola pel que els productes tenen la capacitat de créixer, emetre emissions de CO2 i ser de qualitat imperfecta.

Resumen

Este trabajo de final de grado busca crear un modelo de gestión de inventarios en formato EOQ (orden económica de cantidad) que optimice el ciclo de inventario y el precio final de los productos. Está adaptado a las necesidades de la industria agrícola y ganadera por lo que los productos tienen la capacidad de crecer, emitir emisiones de CO2 y ser de calidad imperfecta.

Abstract

This final project aims to create an inventory management model in EOQ (economical order quantity) format, optimizing inventory cycle time and final price of the products. It is adapted to the needs of the agricultural and livestock industries, products are able to grow, produce CO2 emissions and have imperfect quality.

0. Table of content

1. Introduction

Whenever a business owner looks into warehouse management there are some problems that are often encountered: how often should they place an order to their vendor and which quantity should they order to minimize their costs. This problem is the baseline of the operational research field Economic Order Quantity (EOQ) models and is a pillar of warehouse management.

These EOQ models take into account the costs of holding the units in a warehouse, the price of setting up an order and the demand for that item. The overall goal is to obtain the frequency and quantities of the orders that achieve the maximum profit for the business owner. In this project the central idea is to provide a more tailored solution of this EOQ model for the livestock and agriculture industry by incorporating some additions to the traditional model.

Table 1. Initial concept of the research

WHAT	An EOQ model
WHAT FOR	To maximized profit
WHY	Because traditional EOQ models do not take into account the particular needs of the livestock and agriculture industries
HOW	By adding the key characteristics and needs of these industries to the EOQ model

Source: Own elaboration

In the last years climate change and pollution have become a key component of businesses. Numerous laws and company policies have been put in place in order to adapt businesses to this growing factor. In particular, the livestock and agriculture industries have been in the media's spotlight because of their environmental footprint. The cost of C02 emissions is a key addition to this EOQ model because it reflects the environmental reality of these industries.

When tailoring a solution for the livestock and agriculture field another main aspect came up: the products in this sector are able to grow over time (as for instance, in the meat industry the animals are growing while in the warehouse farm). This is specially relevant

because the costs change greatly from the original item to the final product and they increase over time as for instance, adult animals will consume more food.
Aditionally, since in these sectors items tend to not be identical, not all of them are perfect. Items will go through an inspection process that determines their quality. Those products that are not in good condition, for example those that are ill, will be sold in a batch instead of receiving the full price. This is done because in real life not 100% of the products are perfect, for example in the cattle industry a business owner will not sell those animals that are ill at the same price as those in perfect conditions.

In traditional EOQ models after the optimization process, the business owner obtains how often an order should be placed to the supplier and the quantity of units it should contain. In this research project the business owner will obtain the data above plus which price per unit should be set to maximize profit. This is commonly referred to as price optimization. In other words, with this model the reader obtains the price and frequency of the setup orders that maximize profit. This is done while taking into consideration the particularities of livestock and agriculture businesses explained before.

2. Relevance of the research

During my Bachelor's, I was fortunate enough to participate in a Study Abroad program where I studied in the Mexican university Tecnológico de Monterrey, known for its engineering programs and operational research. This has been a major influence when choosing the field of EOQ models for my research project. While my degree at Tecnocampus has a quantitative base, I chose to expand my knowledge in this area during my stay in Mexico by choosing advanced quantitative classes and participating in an operational research project. I believe this period of my studies has played a major role when choosing a topic on the engineering side, giving me the confidence and background necessary to develop this research.

In my studies, the weight of economic theory has had a major impact in how I approach research: the ideas behind this model come from an economical perspective, exploring trends and economical principles applied to a logistic problem. The mathematical formulation is the tool used to synthetize these ideas into real-life application, analyzing its mathematical feasibility as well as the quantification of its impact.

The importance of this research in the context of my Bachelor's is relevant because it acts as a convergence point for different areas studied throughout my time in university: logistic problems, environmental impact, economic theory, quantitative methodology, warehouse management and critical thinking.

The design of this model is, consequently, not a purely quantitative improvement. It takes into account the interests of the industry while keeping up with the business trends shaped by the different actors of the market. In essence, the added value of using my major as the basis for the model is being able to understand the qualitative needs of warehouse optimization and using my background in logistics to provide a solution interesting at a business level.

Incorporating the Polluters-Pays Principle to the research responds to this economic frame. It represents and arising business trend and adds socio-economic perspective to the research. When a company does not take into account the cost of environmental impact, the costs of production do not consider the cost this impact represents for society, thus creating what in economic theory is known as a negative externality.

The Polluters-Pays Principle is grounded in this negative externality principle, in which the cost of the environmental impact is absorbed by those producing it (and often transmitted to the consumer). The addition of emission costs is used to represent the cost these have for society, a cost that would otherwise be ignored by the company. Administrations tend to represent this cost by adding taxes to carbon emissions, acting as a pigouvian tax. For many industries today, not taking into account carbon restrictions in an inventory model means not providing accurate data for decision-making and using a price that does not reflect true costs. Often, the absence of carbon restrictions implies allocating the weight of emission costs on society (creating negative externalities), hence the social relevance of this research project.

Aside from the addition of carbon taxes by administrations, the private sector has also been involved in the implementation of carbon restrictions. There is a consistent trend in business to adopt a monetary price for carbon emissions, in 2015 alone the number of companies using this concept increased by 23% (World Economic Forum. & N. Bartlett, 2016). This research project incorporates carbon restrictions because it aims to represent this business trend as well as the tendency of incorporating carbon emission taxes in public policy. From a social perspective, the spread of this tendency contributes towards a more sustainable industry while shifting investment towards low-carbon technologies (G. Mestrallet, 2015). In the words of Philippe le Houérou, Chief Executive of the International Finance Corporation: *"Imposing a price on carbon sends a financial signal to investors that low-carbon investments are valuable today and will be even more valuable in the future."*

In conclusion, considering carbon emissions in the EOQ model is relevant since it involves the different actors of the market: suppliers, consumers, investors, the state and the average citizen.

From a purely business perspective, the adoption of carbon constraints is relevant because it considers the reality of CO2 taxation in which governments tax companies based on their CO2 emissions. The revenue generated by pricing carbon emissions as of 2017 is over the 33 billion dollars according to World Bank. Hence, industries who have a high volume of CO2 emissions are likely to experience a larger impact from this taxation in their cost structure. Public policy is linked to the use of carbon pricing by companies, for instance those companies operating under a jurisdiction with carbon restrictions are seven times more likely to use carbon pricing (*State and Trends of Carbon Pricing 2020*, 2020). Nonetheless, public policy is not the only catalyzer, In World Bank's State and Trends of Carbon Pricing 2020 report, internal carbon pricing of companies was stated to be used as a tool for driving energy efficiency and improving internal behavior in more than 50% of companies using carbon pricing. This statement is especially relevant because it demonstrates that incentivizing low-carbon decisions is not purely encouraged by public policy, it is also interesting at a business level.

According to the Food and Agriculture Organization of the United Nations, the C02 emissions from supply chains of the livestock industry accounts for 14.5% of all anthropogenic GHG emissions (Organización de las Naciones Unidas para la Alimentación y la Agricultura, 2006). The idea of incorporating growing items to the model is to provide a solution that fits the needs of this industry.

At an institutional level, the New Circular Economy Plan published by the European Commission (*New Circular Economy Strategy - Environment - European Commission*, 2019) is an example of how administrations are incentivizing industries to reduce waste and design supply chains that consider waste management. From a business owner stand point, the use of circular economies is often linked to better financial performance like McKinsey's 2017 article on the topic suggests (*Mapping the Benefits of a Circular Economy*, 2017).

The socio-economic relevance of this research appeals to the main themes mentioned in the precedent paragraphs: the polluter-pays principle, the environmental impact of the livestock and agriculture industries, carbon pricing and circular economies. These justify the research topic from a socio-economic perspective but might be less clear on pointing out the relevancy of such research in its current time and space frame. The United Nation's (UN) 2030 Agenda for Sustainable Development, adopted in 2015 by all Member States, will be used to justify the importance of this project in relation to global needs in the next decade.

The sustainable development goals (SDG) (*The Sustainable Development Agenda – United Nations Sustainable Development*, n.d.) consists of 17 goals, three of which can be directly related to this project:
- Goal 12, Responsible consumption and production: around a third of all food produced globally is wasted, 13.8% of food being lost in supply chains and the use of natural resources is still used unsustainably (*Sustainable Consumption and Production – United Nations Sustainable Development*, n.d.). Utilizing in the research project the consideration that a percentage of the products is of imperfect quality, is done in order to take into account waste management during supply chain processes, particularly warehouses.

- Goal 13, Climate action: the last decade 2010-2019 has been the warmest decade ever recorded, affecting more than 39 million people in 2018 alone, the emission of greenhouse gases like CO2 being the main root cause (*Climate Change – United Nations Sustainable Development*, n.d.). The addition of carbon restrictions in the model aims to represent this reality by taking into account the impact of CO2 emissions in the decision-making.

- Goal 15, Life on land: human activity has modified 75% of Earth's surface, biodiversity has reduced over time and deforestation continues to decline (*Forests, Desertification and Biodiversity – United Nations Sustainable Development*, n.d.). As mentioned before, the livestock and agricultural sectors are one of the top industries affecting deforestation. In this model, the goal is to contribute towards decreasing the environmental impact of these industries by improving its efficiency, introducing concepts from circular economies, polluting constraints and reducing negative externalities.

The industry and socio-economic tendencies explained in this chapter are key because most operational research, especially in the economic order quantity (EOQ) field has been developed through an industrial engineering perspective, proof being that most of the bibliography for this project comes from industrial engineering journals. It is particularly common in the EOQ field to use the mathematical side of the research as the foundation to improve this model. My research journey has been highly influenced by the theoretical economic framework and business perspective acquired during my education at Tecnocampus. The study of economy, business management and logistic trends learnt during my Bachelors', has served as the main line for the development of this EOQ model.

3. Personal motivation

When I started to think about the research theme I decided to choose a subject challenging at an academic level, I wanted something slightly outside of my comfort zone, hence the important weight of mathematics in my project. My personal motivation is embedded in including my personal vision of the future of supply chains by tackling some of the issues that I have encountered during my studies and my professional career.

I have worked in warehouses since I was eighteen and I have experienced first-hand waste management systems in warehouses (and often, the lack of them). Aside from my personal experience, as stated in the scientific relevance part, warehouses and supply chains are still a key area of waste generation. When I started this EOQ, I knew I wanted to take into account this waste as part of the production process, because not doing so would mean not representing the reality I live day in and day out: waste exists and it is a logistic issue. After this first encounter, I dived deeper into the concept of circular economies, discovering that it resonates with my environmental values and fits the vision I personally have of the future of supply chains.

During my studies at Tecnocampus, I was exposed to the environmental impact of supply chains and its effects on society, especially in terms of CO2 emissions. In my professional life, I often find myself justifying warehouse projects in terms of safety, monetary costs or operational practices but I rarely use CO2 emissions or environmental impact, mainly because it has not been incorporated into my decision-making process.

100 companies are responsible for 71% of all global emissions (*Corporate Honesty and Climate Change: Time to Own Up and Act | NRDC*, n.d.). In my personal life, I often base part of my decision-making in the environmental impact and I wanted to translate this environmental weight into my work field as well. Considering the much larger weight industries have towards climate change when compared to individuals, it made me think about the importance of incorporating this ecologic conscience in my own professional life too.

In my professional career, I aspire to work in a sector that factors in the implication its activity has for the planet, using carbon constraints is an increasing popular way to take this into account. Adding CO2 emissions to the model represents both the increasing reality of CO2 emissions' impact and my own personal values.

In the scientific relevance part, the environmental impact of livestock and agriculture is highlighted. When I chose to incorporate growing items in the model, I wanted to tailor the EOQ model to the needs of this sector in particular because of its large participation towards climate change. When facing this project I wanted to study a topic that could shine some light on these industries, hoping to provide an interesting solution.

For me, climate change is much more than environmental impact or CO2 emissions; it has devastating social effects, especially for women and children in at-risk situations. In the scientific relevance portion social effects are further described, tying them to my personal motivation is probably the easiest part of the thesis: I strongly believe that scientific progress goes hand in hand with social endeavor and as such, it is my duty to give back to the community whenever possible, in my research journey.

My main motivation when deciding to start a scientific research project with a focus in operations was the opportunity to contribute to the scientific community, it is no secret that even today, the gender gap in research and academia is very significant; in Science, Technology, Engineering and Mathematics (STEM) disciplines women represent less than 15% of the total of active authors and publications from male scientist are quoted up to a 30% more (Junming Huang, A. J. Gates, R. Sinatra, & A. L. Barabási, 2020).

Le seul véritable voyage, le seul bain de Jouvence, ce ne serait pas d'aller vers de nouveaux paysages, mais d'avoir d'autres yeux, de voir l'univers avec les yeux d'un autre, de cent autres, de voir les cent univers que chacun d'eux voit, que chacun d'eux est; (M. Proust, 1923, p. 71)

To me, scientific research is about broadening the available knowledge and opening the field for others to do the same. Personally, a field cannot be improved when it is continuously looked from the same lenses, deep devotion to one's field is then necessarily linked to bringing as many perspectives and human experiences as possible. The opportunity cost of not incorporating diversity in operational research is so vast it is impossible to call oneself a scientist and keep the door shut for others.
As a woman in STEM, I see this project as more than just an opportunity to contribute to the scientific community. It is an opportunity to increase the representation of women in operations. It motivates me to contribute towards the opening of the field for those who have a hard time finding a scientist that looks like them, for I was once in their shoes.

I believe that scientific research has to come from a place of strong commitment to the field one is studying along with a deep appreciation for the work of those who came before. Today, it humbles me to be able to write down my own name, whereas others had to rely on a pseudonym, a husband's name or become another anonym lost in womanhood.

4. Research Goals

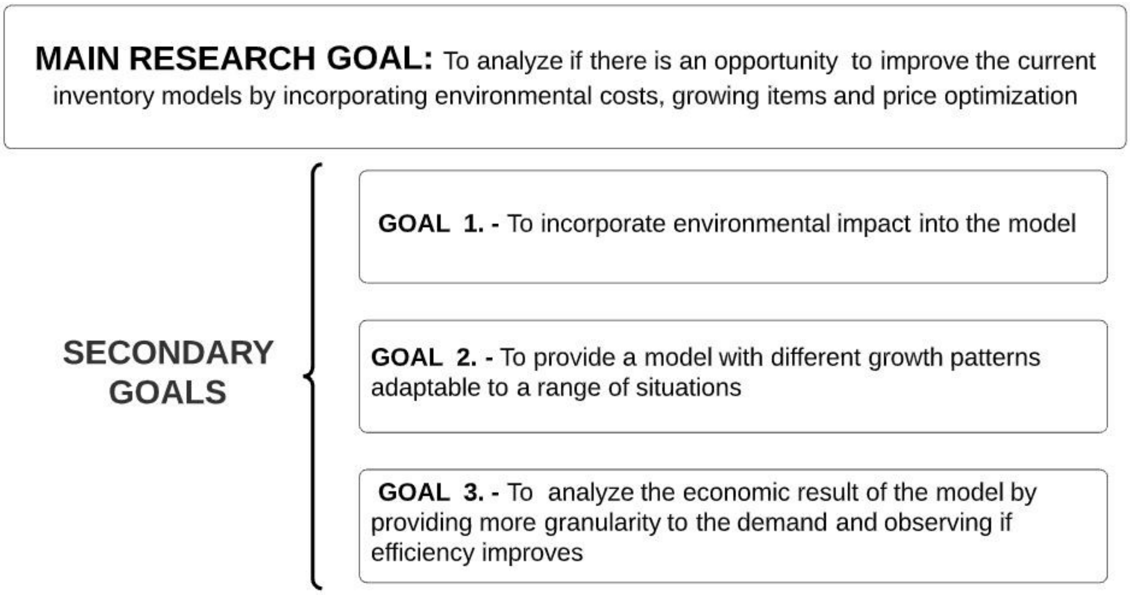
The main goal of this research project is to analyze if there is an opportunity to improve the current EOQ inventory models by incorporating environmental costs, growing items and price optimization. These goals are in turn, aligned with the needs of the livestock and agriculture industries.

In the environmental-related aspect of this project, the goal is to incorporate the cost of the emissions in the cost structure of the model. The goal being; allowing business owners and the different stakeholders to base their decision-making process in a quantitative result that takes into account the weight of environmental impact.

When developing the growth pattern of the model, the aim is to present a solution that is specific enough to provide an accurate solution true to reality but with a computational weight, that facilitates its adaptation to a range of different products and scenarios. Meaning, different products will have different growth patterns so the model should be adaptable to different types of growing items.

The purpose of using demand optimization is to improve the economic result of the inventory model by adding more granularity to the demand. The goal of providing a more flexible mathematical representation of the demand is to facilitate the adaptation of the model to the particular case of the reader, thus, improving its accuracy. The baseline is to provide business owners with more tools to adapt the model to the type of demand they experience.

Graph 1. Research project main goals and secondary goals.



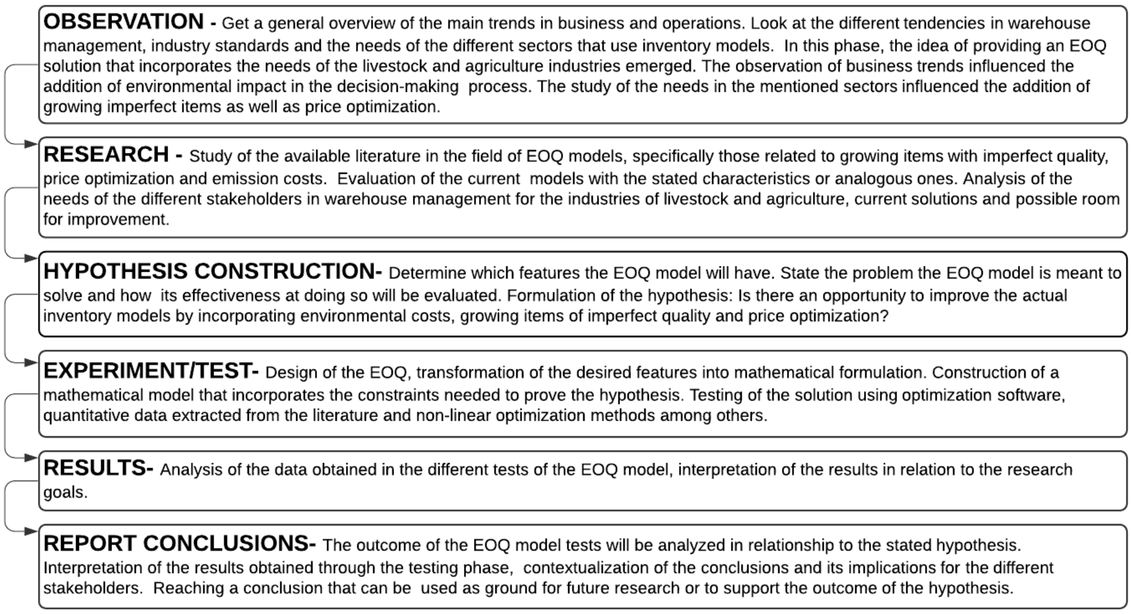
Source: Own elaboration.

5. Methodology
5.1 Methodology overview

The main guideline when approaching this research project is the scientific method Graph 2 shows the systematic process of the scientific method applied to this research. The main process is to construct a model that reflects the needs of a real industry. To do so, literature on the topic will be researched and then the mathematical formulation will be done. To prove if the model is indeed able to solve the problem it will be tested. From the test results and the original research goals, conclusions can be obtained.

This diagram (graph 2) is meant as a summary of the methodology used in relationship to the scientific method but further explanation and details are provided in the following paragraphs.

Graph 2. Scientific method: process breakdown for the research project



Source: Own elaboration.

Because of the scientific nature of this project and the specific field of study, the foundation of the research is based on the family of demand models known as Economic Order Quantity (EOQ). The research of current literature is done in order to provide a solid framework for the development of the model. This framework will also be referenced to study how the different particularities of the model can be incorporated. Lastly, the available literature in the field will be used to determine how to approach this model, so that it can be scientifically relevant.

The mathematical methodology is mainly done through the utilization of non-linear optimization methods. The optimization will be based on a total profit equation, representing the economic impact of the different variables on the total profit. The baseline of the mathematical work is to find the values of time duration of the inventory cycle and price per unit that maximize the total profit.

Figure 1. Mathematical methodology overview

1 Elaborating the total profit function

- A simplified version would be: Total income - Total costs = Total profit
- It incorporates the particularities of the model: growing items, imperfect products, emission costs and demand granularity
- It is an adaptation of the total profit function used in traditional EOQ models

2 Optimizing the total profit function

- The Total profit function will be derived (using a Hessian matrix) to obtain the optimized function.
- This Hessian function is where the different values of each variable will be plugged in.
- Once the values are plugged in, the optimization process (the Hessian matrix) will give the values of total cycle length and price that maximize the total profit function, obtaining the highest possible profit for the business owner.

3 Result of the mathematical process

- After the optimization process, the model can be used
- In this phase the model will be tested to check its effectiveness at achieving the research goals
- The model is ready to be programmed in an optimization software.

Source: Own elaboration.

The optimization is done firstly by developing the total profit equation of the classic EOQ model and incorporating the particularities of this research project: growing items with imperfect quality, price optimization and carbon restrictions. The addition of these traits will be done by taking into account the available literature on the matter and adapting it to the actual model formulation, further explained in the model construction chapter. Carbon restrictions will be expressed in the model by calculating the cost of the tones of C02 produced by the inventory. Growing items attempt to express the particularity of the inventory in the livestock and agriculture sector in which inventory weight varies over time; this will be incorporated in the model using item growth functions from the literature, mainly those referred in Makoena Sabatjane and Adentuji's 2019 paper.

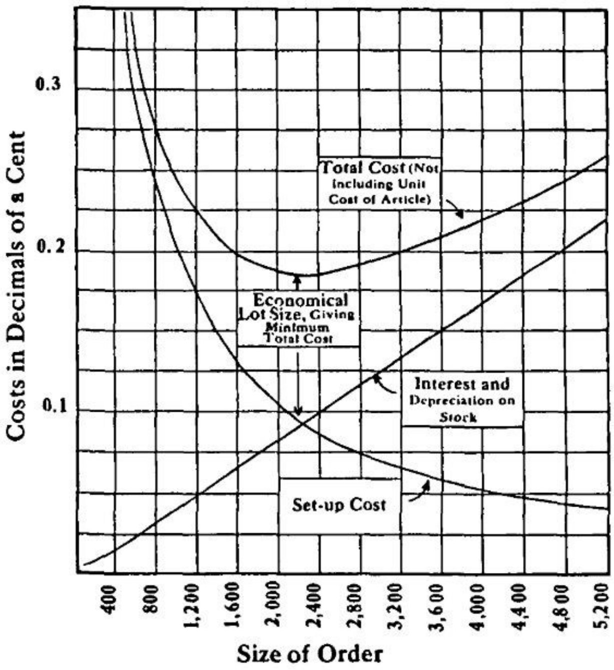
In this research, the optimization from Makoena Sabatjane and Adentuji's 2019 has been replicated from scratch, meaning all optimization and mathematical model construction has been deconstructed and built from zero so that the quality of the base model could be corroborated.

In order to verify the formulation and calibrate the model, the optimization software Lingo will be used. The final mathematical model will be programmed in Lingo where it will test its results against a set of examples provided in papers from the EOQ literature branch (will be specified in the upcoming versions of the document), especially the quantitative examples used in Makoena Sabatjane and Adentuji's 2019 paper regarding growing items. This will be done for two main purposes: to test that the optimization of the total profit function with the different growth functions is correct and to study if adding price optimization does indeed improve the total profit.

6. Theoretical framework: the economic order quantity (EOQ) model

The economic order quantity was first introduced by Ford Whitman Harris in 1913 in his paper "How many parts to make at once" (F.W. Harris, 1913). The EOQ model did not become widely popular until 1988 when it was rediscovered and has been quoted in numerous inventory related research papers (D. Erlenkotter, 1990). Harris' original inventory model finds the optimal quantity of units per lot to order. His first model considered the costs of order set up and the cost of maintaining inventory. The higher the number of units per order the lower the set-up costs, consequently inventory holding costs are higher.

Graph 3. Manufacturing Quantities Curves. Optimal lot size is given by balancing the trade-off between set-up costs and costs of maintaining stock.



Source: Ford W. Harris, *How many parts to make at once* (1913) (Fig.1)

6.1 Growing items and the economic growing quantity (EGQ)

The research of growing items in inventory models is considerably new and scant, Rezaei (J. Rezaei, 2014) was the first work published on the topic. The idea to incorporate growing items to an inventory model is occasioned by the need of certain sectors, such as the livestock or the horticulture industries, to have a more authentic representation of the costs in their inventory modelling.
The main difference between a growing item and a standard item is that the former increases its overall total weight throughout the duration of an inventory cycle. This is also the case for Rezaei's (J. Rezaei, 2014) paper and the main difference with the original EOQ model by Harris (F.W. Harris, 1913). An addition to the original EOQ model is the requirement to include feeding costs; this is due to items needing to be fed in order to grow. In other words, inventory costs are linked to the growing cycle, the bigger the product the more costs it will incur.

The branch of growing items has grown ever since, Zhang et al. (Y. Zhang, L. Y. Li, X. Q. Tian, & C. Feng, 2016) incorporated carbon-constrictions and Khalilpourazari and Pasandideh (S. Khalilpourazari & S. H. R. Pasandideh, 2019) used a multi-item multi-constrained EOQ model. Sebatjane and Adetunji (Makoena Sebatjane & O. Adetunji,

2019) introduced incremental quantity discounts to the growing items original EOQ model while Sebatjane (M. Sebatjane, 2019) analyzed the quantity of items per cycle (lot sizing) through a set of deterministic models.

Nobil et al. (A. H. Nobil, A. H. A. Sedigh, & L. E. Cárdenas-Barrón, 2019) followed linear functions to estimate the growing pattern of the items and the model allowed shortages. The Economic Growing Quantity (EGQ) used in Gharaei and Almehdawe (A. Gharaei & E. Almehdawe, 2019) used numerical examples from the poultry industry. The model included a growth function for items dying during the inventory cycle, incurring in costs for the dead items' disposal and considered different setup costs for dead and alive items.

Following up the literature on case studies of growing items, Malekitabar et al. (M. R. Malekitabar, M., Yaghoubi, S., & Gholamian, 2019) utilized a two echelons supply chain, with one supplier and one farmer and considering the specific mortality rate and growth function for a field in particular: rainbow trout farms.

The issue that is often encountered when facing an inventory model with growing items is that the growth model is hard to adapt to the different items. Sebatjane and Adetunji (Makoena Sebatjane & O. Adetunji, 2019a) work is especially relevant to this issue because it offers a polyvalent model with 3 separate growth models: logistic, linear and split linear growth. The later also includes three different modalities depending on where the target slaughter weight is allocated within the growth region: initial slow growth region (case 1), intermediate fast growth (case 2) and final slow growth region (case 3).

6.2 Imperfect quality items in inventory models

The idea that items that undergo a manufacturing process can have different levels of quality, was incorporated to the traditional EOQ (Economic Order Quantity) model for the first time in 2000 by Salameh and Jaber (M. K. Salameh & M. Y. Jaber, 2000). The inventory model used in Jaber's paper (M. K. Salameh & M. Y. Jaber, 2000) acknowledged that a percentage of the products in each lot was not of perfect quality. In the following years, this inventory model was modified and improved such as in Cárdenas-Barrón (S. K. Goyal & L. E. Cárdenas-Barrón, 2002) whose work modified some computational mistakes made in the expressions of Salameh's (M. K. Salameh &

M. Y. Jaber, 2000) paper. Goyal and Cárdenas-Barrón suggested the computing of the EOQ through a simpler process. Maddah and Jaber (B. Maddah & M. Y. Jaber, 2008a) did the same for the expected total profit.

In recent years, there has been many additions to the work of Salameh and Jaber that vary the approach and extension of the model. Chan (P. B. Chan, W. M., Ibrahim, R. N., Lochert, 2003) integrated lower pricing, rejects and reworks situations to the model. Chang (Hung Chi Chang, 2004) incorporated a fuzzy variable for both the demand rate and the fraction of imperfect quality products and applied fuzzy sets theory to an EOQ with imperfect quality products. Yu et al. (J. C. P. Yu, H. M. Wee, & J. M. Chen, 2005) integrated partial backordering and degradation of the products. Papachristos (S. Papachristos & I. Konstantaras, 2006) analyzed the effectiveness to prevent shortages on the original paper by Salameh and Jaber (M. K. Salameh & M. Y. Jaber, 2000) and included in the EOQ model the ability to withdraw imperfect products.
Wee et al (H. M. Wee, J. Yu, & M. C. Chen, 2007) allowed shortages in the model, Jaber et al. (M. Y. Jaber, S. K. Goyal, & M. Imran, 2008) subjected the original model to learning effects, were the total number of imperfect products was subject to learning effects. Maddah and Jaber (B. Maddah & M. Y. Jaber, 2008b) also reviewed the original EOQ model by Jaber (M. K. Salameh & M. Y. Jaber, 2000) simplifying the model and adding unreliable supply and a screening process.

Chung et al.(K. J. Chung, C. C. Her, & S. Der Lin, 2009) presented an inventory model for items with imperfect quality in a two-warehouse situation. Yoo et al. (M. S. Yoo, S. H., Kim, D., Park, 2009) developed a model that considered two-way inspections and sales return. Hsu and Yu (H. F. Hsu, W. K. K., Yu, 2009) added the ability to acquire one-time-only discounts to the original inventory model. Chang and Ho (Hung Chi Chang & C. H. Ho, 2010) used differential calculus to derive a EOQ model that included imperfect quality products and shortages.
Chen and Kang (L. H. Chen & F. Sen Kang, 2010) further developed the vendor-buyer model by incorporating products of imperfect quality under the conditions of permissible delayed payments.

The EOQ model by Salameh and Jaber (M. K. Salameh & M. Y. Jaber, 2000) is constantly-evolving, for instance Wahab and Jaber (M. I. M. Wahab & M. Y. Jaber, 2010) added different holding costs and learning effects to the earlier version of the EOQ developed by Salameh and Jaber (M. K. Salameh & M. Y. Jaber, 2000). Chang (H. C. Chang, 2011) commented the addition of quantity discounts to the model using applied mathematical modelling. Khan and Jaber (M. Khan, M. Y. Jaber, & M. I. M. Wahab, 2010) used learning inspection and Lin (T. Y. Lin, 2010) applied the mathematical modelling using quantity discounts.

Hsu (H. F. Hsu, W. K., & Yu, 2011) studied the EOQ for imperfect quality products when a price increase is announced. Khan and Jaber (Mehmood Khan, M. Y. Jaber, & M. Bonney, 2011) incorporated inspection errors and the same authors Khan and Jaber (M. Khan, M. Y. Jaber, A. L. Guiffrida, & S. Zolfaghari, 2011) described the current state of published literature regarding EOQs that take into account imperfect quality items. Wahab et al. (M. I. M. Wahab, S. M. H. Mamun, & P. Ongkunaruk, 2011) expanded the inventory system with a two-level international supply chain that also considered environmental impact.

Sadjadi et al. (S. J. Sadjadi, S. A. Yazdian, & K. Shahanaghi, 2012) incorporated the effects of marketing plans and pricing in a model that included products with imperfect quality. Cárdenas-Barrón (L. E. Cárdenas-Barrón, 2012) added discounts per quantity and varying holding cost depending on the quality of the products to a closed-form solution of an inventory system for products of imperfect quality. Rezaei and Salimi's (J. Rezaei & N. Salimi, 2012) inventory model considered inspection shifts from buyer to supplier and imperfect quality to obtain the optimal economic order quantity and purchasing price. Jaber and Zanoni (Mohamad Y. Jaber, S. Zanoni, & L. E. Zavanella, 2013) applied the imperfect quality concept to an entropic EOQ model.

Paul and Wahab (P. Paul, S., Wahab, M. I. M., Ongkunaruk, 2014) used joint replenishment and price discount to develop an inventory model with imperfect quality products. Zhou and Chen (W. Zhou, Y. W., Chen, J., Wu, Y., Zhou, 2015) added the possibility of one-time-only discounts. Modak and Panda (N. M. Modak, S. Panda, & S. S. Sana, 2015) used imperfect quality items to optimize a preventive maintenance just-in-time buffer inventory. Wang et al. (W. T. Wang, H. M. Wee, Y. L. Cheng, C. L. Wen, & L. E. Cárdenas-Barrón, 2015) incorporated a constraint regarding screening and partial backorders to an EOQ model for imperfect quality products. In Taleizadeg et al. (A. A. Taleizadeh, M. P. S. Khanbaglo, & L. E. Cárdenas-Barrón, 2016) the pricing and ordering decisions were made through a model that considered the use of inspection and two

types of items: those with imperfect quality and those defective, the latter were subject to buyback. In Rezaei (J. Rezaei, 2016) the same inspection process was utilized but in this case it was done under sampling inspection plans for imperfect items.
Other authors further developed the management of inventories with items that can be of imperfect quality, for instance in Khan and Jaber (Mehmood Khan, M. Y. Jaber, S. Zanoni, & L. Zavanella, 2016) the supply chain has defective items and the stock is managed by the vendor through a consignment agreement. Alamari et al (A. A. Alamri, I. Harris, & A. A. Syntetos, 2016) followed an efficient approach to the imperfect quality items constriction. Shekarian and Olugu (N. Shekarian, E., Olugu, E. U., Abdul-Rashid, S. H., Kazemi, 2016) subjected different holding costs to fuzziness and learning, considering that as time goes by workers and systems improve their performance because they gain experience and knowledge.

In recent years, the literature about imperfect quality items has expanded in many directions, adding to the classic EOQ model for imperfect quality items new restrictions taking into account the new tendencies in the industry. Lately, the upswing of anti-climate change policies targeting emissions has increased the number of works in the field that take into account carbon tax policies in the mathematical models. Lin and Sarker (B. R. Lin, T. Y., Sarker, 2017), Tiwaris, Darynto and Wee (S. Tiwari, Y. Daryanto, & H. M. Wee, 2018) and Kazemi et al. (N. Kazemi, S. H. Abdul-Rashid, R. A. R. Ghazilla, E. Shekarian, & S. Zanoni, 2018).

The latest research in the field broadens towards different branches, Rad and Khoshalhan (C. H. Rad, M. A., Khoshalhan, F., Glock, 2018) analyzed both the two-stage supply chain and price-and-advertisement sensitive demand with imperfect items. Sebatjane and Adetunji (Makoena Sebatjane & O. Adetunji, 2019a) fusion the growing items field with the imperfect quality research. Their work is especially interesting because it incorporates both concepts in the model and optimizes three EOQ models where the items in each model follow a different growing pattern. Thus, obtaining the optimal duration of an inventory cycle taking into account the classic EOQ restrictions, the growth pattern, the imperfect quality inspection process and the ability to discriminate the selling price depending on the quality.

6.3 Carbon emissions restrictions

The EOQ model has changed over time in order to adapt to new tendencies and industry changes. Incorporating environmental impact to inventory models has experienced a rapid increase in recent years, especially with the spread of climate change awareness and the raise of environmental policies against warehouse emissions.

The development of environmentally responsible inventory models by Bonney and Jaber (M. Y. Bonney, M., Jaber, 2011) followed the idea that if a cost charged does not reflect the true environmental cost, the decisions making process is done based on erroneous data. Hua and Cheng (S. Hua, G., Cheng, T. C. E., Wang, 2011) and Benjaafar et al. (M. Benjaafar, S., Li, Y., Daskin, 2012) incorporated carbon footprints, the former used an EOQ model while the later provided different insights from simple models.
Rosič and Jammernegg (W. Rosič, H., Jammernegg, 2013) presented a dual sourcing model taking into consideration both economic and environmental performances. Incorporating environmental impact has caused many researchers to revisit industry models from a sustainability approach. For instance, Arslan and Turkay (M. (2013). Arslan, M. C., Turkay, 2013) introduced sustainability considerations to the classic EOQ model, Chen, Benjaafar et al. (A. Chen, X., Benjaafar, S., Elomri, 2013) used carbon constraints for the inventory model and Absi et al. (Absi, N. et al, 2013) took into account carbon emission constraints in the lot sizing. Jaber et al. (A. M. Jaber, M. Y., Glock, C. H., El Saadany, 2013) adopted an user based approach where incentives were placed to reduce emissions in a two-level supply chain with GHG emissions costs and penalties for exceeding the maximum amount.
The research field of inventory models has been experiencing a shift towards models that incorporate environmental impact as a cost to be considered when analyzing and constructing operational models. Bozorgi et al. (D. Bozorgi, A., Pazour, J. Nazzal, 2014) considered emissions as part of their cold items inventory model, Battini et al. (F. Battini, D., Persona, A., Sgarbossa, 2014) presented theoretical formulations and applications of the sustainable EOQ model. Bozorgi (A. Bozorgi, 2016) expanded the multi-product inventory field by using cold items and emission consideration in the model.

In the imperfect quality items research field, the tendency to take into account environmental impact has also been increasing over time. Tiwari et al. (S. Tiwari, Y. Daryanto, & H. M. Wee, 2018) formulated an inventory management with both deteriorating items and carbon emissions considerations. Kazemi et al.'s (N. Kazemi, S.

H. Abdul-Rashid, R. A. R. Ghazilla, E. Shekarian, & S. Zanoni, 2018) paper is one of the most recent pieces of literature in the field of imperfect quality emissions also considered emissions.

6.4 Price optimization in EOQ models

Price optimization is a fairly extended practice in decision models. In this variation of the classic EOQ model (Ford W Harris, 1913) takes into account the impact of price variation in demand and total profit. This is often done by establishing a relationship between price and demand, optimizing the price by finding the price which will give the maximum total expected profit within the conditions of the particular EOQ model.

Price optimization was popularized in the eighties and early nineties, influenced by the success of Japanese companies when integrating marketing and production (J. Eliashberg & R. Steinberg, 1993). Eliashberg and Steinberg in Chapter 18 of their article further explain the early works in this branch of operational research and its role as an interface between marketing and production.

While in the original EOQ models price is given as constant, in this research project, price is treated as a decision variable for the business owner, hence, literature framework will mainly focus on this branch. Whitin (T. M. Whitin, 1955) was the first to incorporate price-dependent demand to the EOQ model, Arcelus (F. J. Arcelus & G. Srinivasan, 1987) further explored this branch by using a replenishment policy in the EOQ.
For the mathematical representation of price optimization, demand is considered to be sensitive to price. Huang et al. (Jian Huang, M. Leng, & M. Parlar, 2013) does a great job at breaking down the most popular demand models, it provides further information on demand functions as well as the most relevant literature on the topic. As described in this same paper, price-dependent demand models are the most common because they are the most effective at representing a firm's impact on demand (Jian Huang, M. Leng, & M. Parlar, 2013).

The demand function that will be used in this research is a power model (referenced as PM IV in Jian Huang, M. Leng, & M. Parlar, 2013 paper's). The original function appeared in Chen's inventory control paper (Y. Chen, S. Ray, & Y. Song, 2006). While the linear model is very popular due to its practicality to represent demand functions, its assumptions often need an upper bound on price, which in most cases does not represent reality. The choice of using a power model solves this issue plus it is capable of representing both nonlinear and linear patterns, making the model more versatile. Other advantages of using the power model is that it can be linked to the Cobb-Douglas function, traditionally used for utility of production (T. H. Oum, 1989).

6.5 Literature overview

The literature provided in the former chapters gives an in depth framework of the theory and research papers relevant to this branch of research and which have been used to develop this project.

The following table offers a direct comparison between this project and the closest papers from the theoretical framework in terms of scope and characteristics.

Table 2. Current literature that includes a relevant mention of the key topics of this research paper, gap analysis:

Reference	Growing items	Imperfect quality	Carbon restrictions	Price optimization
This research project	X	X	X	X
(J. Rezaei, 2014)	X		X	
(Makoena Sebatjane & O. Adetunji, 2019a)	X	X		
(Y. Zhang, L. Y. Li, X. Q. Tian, & C. Feng, 2016)	X		X	

Source: Own elaboration

7. Model construction
7.1 Model construction general overview

Most EOQ models available today, even those that incorporate C02 restrictions consider its products to be static, meaning they do not experience any change in nature over time. The development of this model is rooted in the lack of literature that takes into account the needs of one of the most polluting industries today: the livestock and farming sector. These industries have the particularity that their products change greatly with age, consequently, changing their weight and C02 production over time. Traditional EOQs, even those that include carbon restrictions, do not contemplate the variation in the characteristics of products nor do they study the effects this can have in the incurrence of carbon emissions and holding costs over time. The aim of this model is to provide a better fit for the calculation of warehouse costs and their environmental impact, in those industries where products experience weight gain over time.

The model takes into account that in most industries there is a slight variation in final items, resulting in a small percentage of the products being of imperfect quality. In the model, this quantity of imperfect items is sold in a batch after the inspection period (in which the company classifies de quality of each product). Incorporating this characteristic in the model is based on the increasing relevance of circular economies where waste, in this case imperfect items, are considered part of the production process. Selling imperfect items in batches also goes hand in hand with the concept used in circular economies, in which waste can be transformed in value added for the business (J. Sánchez-Ortiz, V. Rodríguez-Cornejo, R. Del Río-Sánchez, & T. García-Valderrama, 2020).

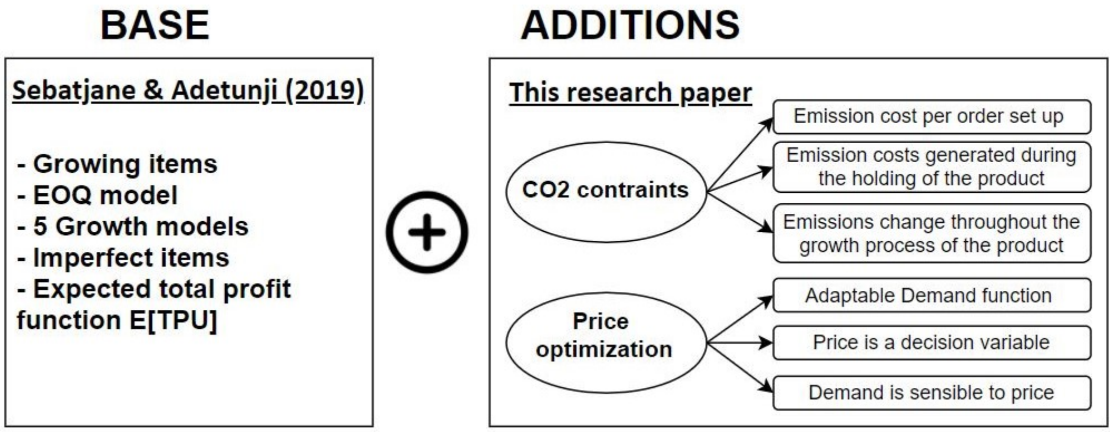
Table 3. Overview of the model and comparison with traditional EOQ models

Traditional EOQ model	EOQ model to be constructed
GOAL: obtains how often an order should be placed to the supplier to maximize profits	GOAL: obtains how often an order should be placed to the supplier and at what price the item should be sold to maximize profit
COSTS include: • Cost of holding an item in the warehouse • Cost of setting up an order	COSTS include: • Emission costs from the setup • Emission costs from holding the item in the warehouse • Holding costs which in this case, change over time (items grow so they consume more over time) • Cost of inspection (not all items are perfect so they are inspected to determine quality)
CHARACTERISTICS OF THE ITEMS: • Do not change in nature over time • Are of perfect quality	CHARACTERISTICS OF THE ITEMS: • They grow over time • A percentage is imperfect
APPLICATION: Most industries that involve a production process	APPLICATION: Livestock and agriculture industries

Source: Own elaboration

This research project is based on Sebatjane's & Adentunji's 2019 paper "Economic order quantity model for growing items with imperfect quality". For the test phase, the data used will be obtained from the quantitative example in this same paper. The following figure explains what additions are done and which aspects belong to the original model.

Figure 2. Overview of the model construction



Source: Own elaboration

7.2 Notations

y = Total number of initial products per cycle

Q_0 = Quantity of initial products per cycle

T = Cycle time

w_0 = Weight per product at initial time

w_1 = Weight per product at time t

w_t = Weight per product at time t

Q_t = Total weight of the products in inventory at time t

p = Cost of buying a unit of weight

s = Price obtained per every unit of weight of a perfect quality product

v = Price obtained per every unit of weight of an imperfect quality product

h = Holding costs per unit time per unit weight

K = Cost of setup per cycle

E = Overall carbon emissions

E^k = Units of Emissions per order order setup (ton)

E^h = Units of Emissions in holding one product (ton)

ε = Tax per unit of emissions (cost/ton)

D = Demand of perfect quality products in weight units

c = Cost of feeding during one unit of time one unit of weight
x = Percentage of the total weight of the products that is of imperfect quality
z = Costs of inspecting one unit of weight
r = Rate of the inspection
t_0 = Starting point, time of the reception of the order, initial time
t_1 = Duration of the growing period
t_2 = Duration of the inspection time
t_s = Duration of the setup time
$E[.]$ = Expected value of a random variable
e = Unit carbon emission (ton/setup order) related to the setup of an order
g = Unit carbon emission (ton/unit hold) related to the holding of one unit in the warehouse
ε = Tax per unit of emissions (cost/ton)
a = Scale parameter for the price dependent demand
b = Sensitivity parameter for the price dependent demand
φ = Power index of demand
α = Asymptotic weight of the products (model I)
β = Integration constant (model I)
λ = Exponential growth rate (model I)
δ_1= Linear growth rate per the first growth region of Model III case 1
δ_2= Linear growth rate per the second growth region of Model III case 2
δ_3= Linear growth rate per the third growth region of Model III case 3

7.3 Assumptions
7.3.1 Assumptions of the EOQ model

The following are assumptions standard to most EOQ models and coincide with those assumptions used in models with analogous characteristics (imperfect quality, emission costs, growing items):

- There is only one type of product.
- During the consumption period holding costs are generated.
- All products regardless of their quality grow at the same rate.
- The inspection procedure does not have inefficiencies.

- All perfect quality items are not sold at the same time, they follow a consumption rate.
- Feeding and the weight-gain of products generates costs.
- Feeding costs are proportional to the weight of the products.
- There is a percentage of the final products that are of imperfect quality.
- Imperfect products can not be fixed or substituted.
- The selling price of good quality items is greater than that of the poorer quality items.

7.3.2 Assumptions particular to this EOQ model

The following are assumptions put in place to adapt the model to its particular characteristics and might not even coincide with those models with analogous features:

- Demand is price dependent and can be non-linear.
- All imperfect products are vended.
- Imperfect quality items are sold altogether in a batch after inspection.
- Cost of emissions (e) and (g) are considered constants.

The following assumptions while they are particular to this EOQ model, they are conditioned by the mathematical formulation and the construction of the Hessian Matrix (further explanation can be found in point 6.6. Optimization model):

- The elasticity of demand and its sensitivity parameters are constants, elasticity does not depend on the location of the demand curve. This particular assumption is added to fit the requirements of the demand model, it is also detailed in the original paper by Jian Huang, M. Leng, & M. Parlar (2013).
- In the demand function (19) variables (a) and (b) are greater than 1, this is restricted by the Hessian Matrix theorem.
- The power index (φ) for the demand function is equal or greater than 1.

7.4 Model development

At the start of the cycle, y quantity of products are acquired with an initial weight of w_0 per product. Each product is able to grow, at the start of the growing period the total weight at the starting point (t_0) is Q_0, obtained by multiplying the quantity of initial products for cycle by the unit weight per product at the start of the growing cycle ($Q_0 = yw_0$). The products, while being fed, grow over time until they reach the optimal weight w_1, then they are inspected and sold. At the time of selling the products the total weight of the inventory is $Q_1 = yw_1$.

Since items can be imperfect, they have to be inspected to determine their quality. The inspection of the products has a duration of t_2 at a rate of r. Graph 4 illustrates the inventory system, where x is the percentage of imperfect quality products. These imperfect products are not sold at a standard price, they are sold in a batch at a lower price. The perfect quality items are sold during the consumption period, T, following the rate of D units of weight per unit of time.

The goal of the inventory model developed in this research project is to maximize the total profit (TP), this is equivalent to the total revenue (TR) minus the company's total costs.

7.4.1 Total profit

The total profit per cycle function is given by six elements: purchasing costs (PC), setup costs (SC), feeding costs (FC), holding costs (HC), inspection costs (IC) and emission costs (EC). The model is designed to maximize the total profit of the company, described as the subtraction of the total costs from the total revenue (TR). The function to calculate total profit per cycle is the following:

(1) $$E[TP] = E[TR] - PC - SC - FC - E[HC] - ZC - EC$$

The expected duration of the cycle is obtained by dividing the total weight of the expected perfect quality products produced during one cycle by the total demand for perfect quality items (in weight units).

(2) $$E[T] = \frac{yw_1(1 - E[x])}{D}$$

Doing the inverse of the function gives the number of cycles that a company should produce in order to fulfill the demand.

The time spent on the mandatory inspection process t_2, depends on the inspection rate (r) and the total weight of the inventory (yw_1). The period in which good quality products are separated from those of imperfect quality, is expressed as period t_2.

(3) $t_2 = \frac{yw_1}{r}$

7.4.2 Expected revenue

The total expected revenue includes revenue generated by selling both perfect quality products and imperfect quality products. After the inspection period, imperfect quality items are sold at a lower price of v per unit of weight in a batch. Perfect quality products are vended continuously at a price of s .

(4) $E[TR] = syw_1(1 - E[x]) + vyw_1E[x]$

7.4.3 Costs

7.4.3.1 Setup cost per cycle

The fixed cost to setup each cycle ($\mathbf{K}$) is incurred at the starting point of each cycle given by:

(5) $SC = K$

7.4.3.2 Cost of purchase

The company buys y initial products per cycle, each initial product weights w_0, and the cost is p per every unit of weight. The purchase cost of the initial products is:

(6) $PC = pyw_0$

7.4.3.3 Cost of inspection per cycle

The total cost of inspection per cycle (ZC) depends on the total weight to be inspected per cycle. The number of monetary units that costs the inspection of one unit of weight is z. The total cost of inspection per cycle is given by:

(7) $ZC = zyw_1$

7.4.3.4 Expected holding cost per cycle

The expected holding cost is calculated considering the quantity of units that will be in the warehouse during a cycle (H). This quantity H is calculated taking into account two factors:

- Consuming time: calculated by dividing the total number of weight of perfect quality to the second power ($y^2 w_1^2 (1 - E[x])^2$) and dividing it by the demand multiplied by 2.

Consuming time = (8) $\frac{y^2 w_1^2 (1 - E[x])^2}{2D}$

- Screening time: Time it takes to scan all the weight in order to classify its quality. This is taken as part of the holding costs because during this time items continue to incur costs.

Screening time = (9) $\frac{y^2 w_1^2 E[x]}{r}$

The holding quantity is obtained by adding the screening and consuming time:

(10) $$\mathrm{H} = \left[\frac{y^2 w_1^2 (1 - E[x])^2}{2D} + \frac{y^2 w_1^2 E[x]}{r} \right]$$

In order to calculate the expected holding cost per cycle $E[HC]$, this quantity of units (H) is multiplied by the monetary costs of holding a single unit for the duration of a cycle (h).

(11) $$E[HC] = h \left[\frac{y^2 w_1^2 (1 - E[x])^2}{2D} + \frac{y^2 w_1^2 E[x]}{r} \right]$$

7.4.3.5 Feeding cost per cycle

The amount of food consumed by the products depends on the weight of each product, depending on the growth function w_t. The growth of items happens during period t_1. The cost of feeding per cycle depends on this variables and the quantity of products bought per cycle (y). Where c is the cost of feeding one unit of weight during one unit of time and $dt$is the number of ordered items on t .

(12) $$FC = cy\int_0^{t_1} w_t dt$$

7.4.3.6 Emission costs
7.4.3.6.1 Setup emissions

Each setup causes emissions of C02, the quantity of emissions produced during this process is calculated by multiplying the emissions that are produced per unit (e) per the quantity of units in the setup.

(13) $$K_{Ec} = e\left(\frac{\mathrm{D}}{\mathrm{yw}_1(1-\mathrm{E}[\mathrm{x}])}\right)$$

7.4.3.6.2 Expected holding emissions

The emissions generated during the holding process are calculated by multiplying the quantity of units hold during a cycle (H) per the quantity of emissions generated per unit on hold (g).

(14) $$\mathrm{H} = \left[\frac{y^2 w_1{}^2(1-E[x])^2}{2D} + \frac{y^2 w_1{}^2 E[x]}{r}\right]$$

(15) $$H_{Ec} = g\left[\frac{yw_1(1-E[x])}{2} + \frac{yw_1 DE[x]}{r(1-E[x])}\right]$$

7.4.3.6.3 Total expected emission costs

For the emission costs function, the model will take into account the processes in the EOQ that produce emissions: the setup of orders and the holding of products.
The expected emission costs [EC] are calculated when multiplying the total quantity of emissions per the monetary cost or tax of one unit of emission (ε). In other words, the cost per unit of emission multiplied by the total quantity of emissions.

(16)
$$EC = \varepsilon\left(e\left[\frac{D}{yw_1(1-E[x])}\right] + g\left[\frac{yw_1(1-E[x])}{2} + \frac{yw_1DE[x]}{r(1-E[x])}\right]\right)$$

Therefore, the formula for EC with the simplified setup and holding emissions functions is the following:

(17) $EC = \varepsilon(K_{Ec} + H_{Ec})$

7.4.4 Expected total profit function

The expected total profit per unit time (per cycle) is found by substituting the expressions of the $E[T]$ equation with the complete equations:

(18)
$$E[TPU] = sD + \frac{vDE[x]}{(1-E[x])} - \frac{pDw_0}{w_1(1-E[x])} - \frac{KD}{yw_1(1-E[x])} - \frac{zD}{(1-E[x])} - \frac{cD}{w_1(1-E[x])}\int_0^{t_1} w_t dt - h\left[\frac{yw_1(1-E[x])}{2} + \frac{yw_1DE[x]}{r(1-E[x])}\right] - \varepsilon\left(g\left[\frac{yw_1(1-E[x])}{2} + \frac{yw_1DE[x]}{r(1-E-[x])}\right] + e\left[\frac{D}{yw_1(1-E[x])}\right]\right)$$

7.4.5 Demand function

This model uses a more complex approach towards demand. While demand is expressed with the D symbol throughout the model development for clarity reasons, demand in this model is made up with the following formula:

(19) $D = a - bs^{\varphi}$

In which demand (D) is composed by the scale parameter (a), the sensitivity parameter (b) and the power index (φ) for the price dependent demand. Adding these parameters to the demand formula allows for an increase in precision when optimizing the model. A further explanation on why this formula was choosen and literature on the topic can be found in section 3.4.The formula of $E[TPU]$ with the full demand breakdown is the following:

(20)

$$E[TPU] = s(a-bs^{\varphi}) + \frac{v(a-bs^{\varphi})E[x]}{1-E[x]} - \frac{p(a-bs^{\varphi})w_0}{w_1(1-E[x])} - \frac{K(a-bs^{\varphi})}{yw_1(1-E[x])} - \frac{z(a-bs^{\varphi})}{(1-E[x])}$$
$$-\frac{c(a-bs^{\varphi})}{w_1(1-E[x])}\int_0^{t_1} w_t dt - h\left[\frac{yw_1(1-E[x])}{2} + \frac{yw_1(a-bs^{\varphi})E[x]}{r(1-E[x])}\right]$$
$$-\varepsilon\left(g\left[\frac{yw_1(1-E[x])}{2} + \frac{yw_1(a-bs^{\varphi})E[x]}{r(1-E-[x])}\right] + e\left[\frac{(a-bs^{\varphi})}{yw_1(1-E[x])}\right]\right)$$

7.5 Growth models

The mathematical model considers the ability of items to growth over time until desired weight is achieved (w_1). The growth pattern of the items has been split into 3 different types of models. The main reason being the ability to cover a wider range of products that follow at least one of these 3 growth patterns.

7.5.1 Model I. Logistic growth function

The logistic growth function is a pattern that can be found in the growth behaviour of numerous products. The growth pattern used in this model is one of the most popular functions when expressing product growth, mainly due to the fact that its depiction of growth is realistic for most items (Makoena Sebatjane & O. Adetunji, 2019a).

This model is settled on the idea that products accelerate the increase of their weight gain over time, until reaching a point where the velocity of the weight gain diminishes .

It should be noted that a product's initial weight is different than zero ($w_0 \neq 0$) therefore, the initial quantity of total inventory weight (Q_0) is not zero either, as graph 4 illustrates. The function showcases an asymptotic behaviour in both ends of the growing period, this is explained by products gaining weight slower in the early stages and slowing down their growth as they near maturity (Q_1). The growth function, weight gain process, of the model being given by:

(21) $$w_t = \frac{\alpha}{1 + \beta e^{-\lambda t}}$$

Products are ready to be sold once they reach target weight (Q_1) in t_1 time. The inspection process takes t_2 , obtained when dividing the total weight of the products (once maturity is reached) by the rate (r) at which products are inspected:

(22) $$t_2 = \frac{y w_1}{r}$$

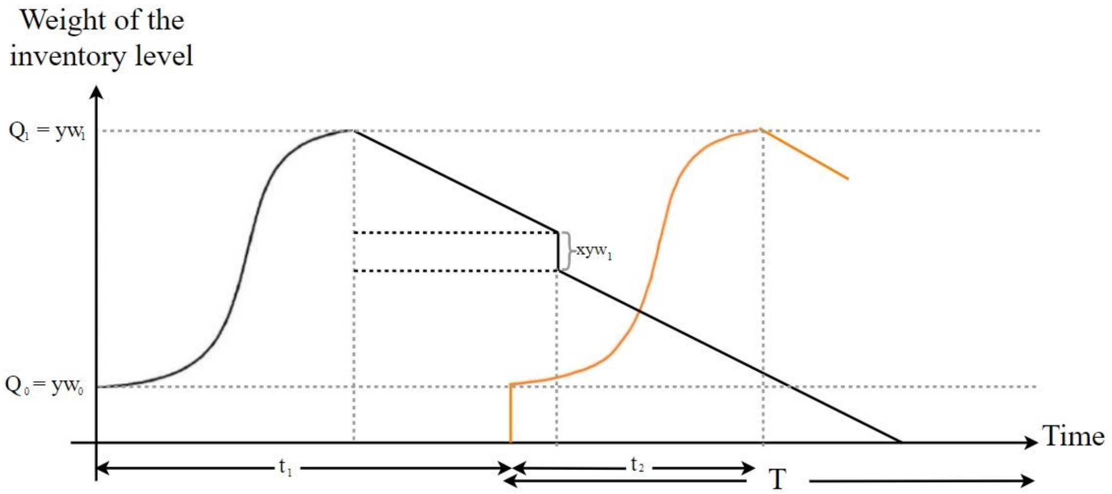
The expected duration of the consumption period per cycle ($E[T]$) is reached when dividing the total weight of perfect products ready to be sold by the demand for perfect products, thus:

(23) $$E[T] = \frac{y w_1 (1 - E[x])}{D}$$

The feeding cost (FC) depends on the weight of the products (which varies over time). Feeding costs are, consequently, computed with the cost of nourishing one unit of weight (c) per the total number of products (y), for the different weights (w_t) products will go through during the growth period (t_1) until target weight is reached (w_1). The feeding costs expressed as:

(24)
$$FC=c\int_0^{t_1} yw_t dt = cy\int_0^{t_1} \frac{\alpha}{1+\beta e^{-\lambda t}} = cy\left[\alpha t_1 + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda t_1}\right) - \ln(1+\beta)\right]\right]$$

Graph 4. Inventory weight over time for model I (logistic growth function)



Source: Own elaboration. Adaptation of Fig.1 "Behavior of an inventory system for growing items with imperfect quality." in Sebatjane and Adentuji's 2019 paper on EOQ model for growing items with imperfect quality.

7.5.2 Model II. Linear growth function

The growth function in this model is transformed into a linear slope, this simplifies the model and diminishes its computational complexity.

In this linear growth model, products increase their weight at a constant rate over time, where the rate (γ) is defined as the number of weight units gained over a unit of time. The model is identical to Model I, the only difference being the weight growth during t_1. Where the growth period is:

(25)
$$t_1 = \frac{w_1 - w_0}{\gamma}$$

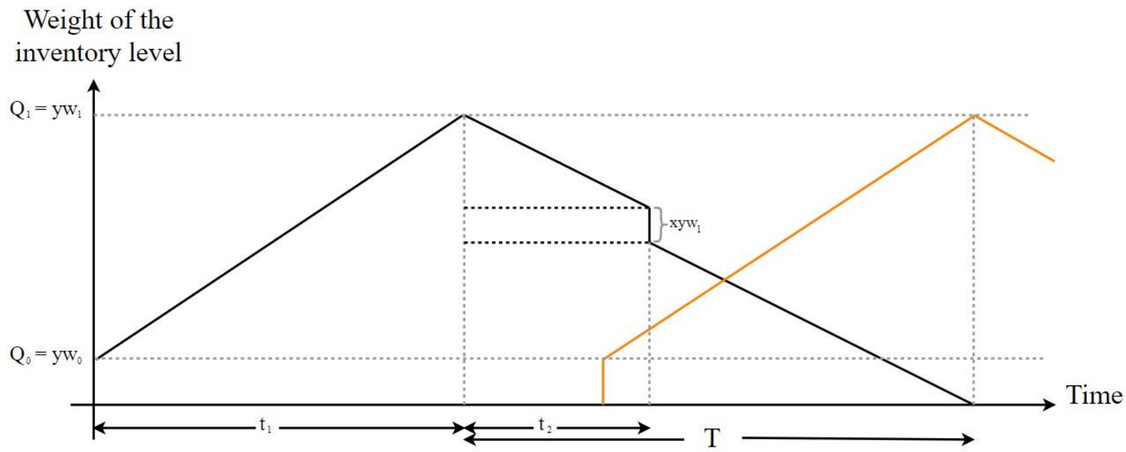
Since the growth period is different, the feeding costs (FC) are calculated using the total weight fed during t_1 .

(26)
$$FC = c\left[\frac{t_1\left(yw_1 - yw_0\right)}{2}\right]$$

For clarity reasons in the equation for feeding costs (FC) eq. (26), t_1 is subtituted by its full formula (Eq. (25)), resulting in the following feeding costs equation:

(27) $$FC = cy\left[\frac{\left(w_1 - yw_0\right)^2}{2\gamma}\right]$$

Graph 5. Inventory weight over time for model II (linear growth function)



Source: Own elaboration. Adaptation of Fig.3 “Behavior of an inventory system with a linear growth function.” in Sebatjane and Adentuji’s 2019 paper on EOQ model for growing items with imperfect quality.

7.5.3 Model III. Split linear growth function

This model is somewhat of an hybrid between the two former ones. It provides a linear growth that approximates the growth function of model I (eq. (24) in model I) thus, simplifying the computational complexity. On the other hand, the growth is split in different sections with different growth rates, which offers a more realistic representation of product growth. This allows the reader to use the version of this model that better fits their particular situation. The growth rate for each of the three regions is expressed as δ_1 ,δ_2 and δ_3 respectively.

This split in 3 different growth areas is done in order to mimic the principle used for model I (Logistic Growth), where products grow slowly at first, reach a fast paced growing rate and decrease their growth rate once they approach maturity. The main difference between the 3 cases provided for this model, is in which moment in time the target weight (w_1) is reached. Each of the growth regions has a particular target weight and time duration, represented as w'_1 and t'_1 respectively for case 1. For case 2 these become

w''_1 and t''_1. Case 3 doesn't have its own notation since both the target weight and the time duration of the cycle are the same as in model 1.

7.5.3.1 Model III. Split linear growth function: Case 1

In this case, the target weight of the products happens in the first growth region. The target weight is given by:

(28) $w_1 = w_0 + \delta_1 t'_1$

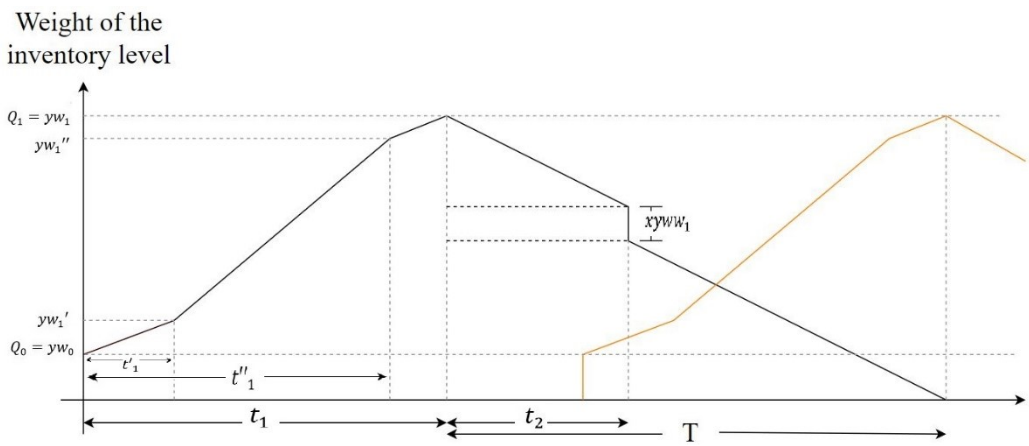
The duration of this growth period is calculated as:

(29) $t_1 = \frac{w_1 - w_0}{\delta_1}$

The expected total profit function $E[TP]$ is the same as in model I and model II, following Eq. (1) the only part affected by model III is the feeding costs equation (FC) since it is the only part affected by growth. This is true for all cases. In case 1 feeding costs are:

(30) $FC = \frac{\left(w_1 - w_0\right)^2}{2\delta_1}$

Graph 6. Inventory weight over time for model III case 1 (split linear function)



Source: Own elaboration. Adaptation of Fig.4. "Inventory system behavior with a split linear growth function" in Sebatjane and Adentuji's 2019 paper on EOQ model for growing items with imperfect quality.

7.5.3.2 Model III. Split linear growth function: Case 2

In this case, the target weight (w_1) is located in the middle region of the growth function, the target weight is given by:

(31) $w_1 = w'_1 + \delta_2 (t_1 - t'_1)$

The duration of the growth period is:

(32) $t_1 = t'_1 + \left(\frac{w_1 - w'_1}{\delta_2} \right)$

The feeding cost per cycle becomes:

(33) $FC = cy \left[\frac{(w'_1 - w_0)^2}{2\delta_1} + \frac{(w_1 - w'_1)^2}{2\delta_2} + \frac{(w_1 - w'_1)(w'_1 - w_0)}{\delta_2} \right]$

7.5.3.3 Model III. Split linear growth function: Case 3

The last case is when the target weight falls in the last growth period. The target weight eq. (34) , growth period eq. (35) and feeding costs eq. (36) are the following, respectively:

(34) $w_1 = w''_1 + \delta_3 (t_1 - t''_1)$

(35) $t_1 = t''_1 + \left(\frac{w_1 - w''_1}{\delta_3} \right)$

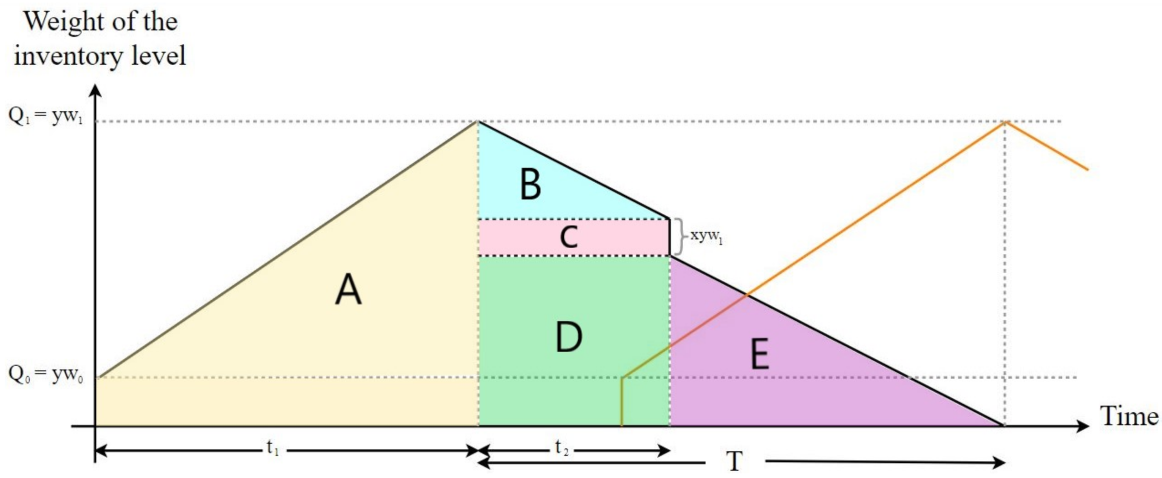
(36) $FC = cy \left[\frac{(w'_1 - w_0)^2}{2\delta_{1w}} + \frac{(w''_1 - w'_1)}{2\delta_2} + (t''_1 - t'_1)(w'_1 - w_0) + \frac{(w_1 - w''_1)^2}{2\delta_3} + \frac{(w_1 - w''_1)(w''_1 - w_0)}{\delta_3} \right]$

7.5.4 Graph representation breakdown

In graph 7, area A represents the total weight sustained over the growing period t_1. Area B reflects the inspecting period (which lasts all t_2) where all the weight is screened and the slope (hypotenuse) of the triangle represents the velocity at which the items are inspected.

In area C the weight diminishes by a discreet amount because all the imperfect items are sold on a batch at the end of the inspecting period t_2. Area D is consequently, the weight of all the perfect quality items sustained over the screening period. Area E is considering all that perfect quality weight hold over the time it takes to consume it, the rate of consumption is represented in the slope of the graph in this section E.

Graph 7. Inventory weight over time graph breakdown (for model III case 1)



Source: Own elaboration. Adaptation of Fig.3 "Behavior of an inventory system with a linear growth function." in Sebatjane and Adentuji's 2019 paper on EOQ model for growing items with imperfect quality.

7.5.4.1 Graph representation breakdown: formulas of the graph

From graph 7, the formulas to calculate the areas and thus the total weight accumulated in each section, are the following:

$$Area\ A = (t_1 Q_0) + [(Q_1 - Q_0)(t_1)]/2$$

$$Area\ B = [(Q_1 t_2) - (Area\ D + (xyww_1 t_2)]/2$$

$$Area\ C = xyww_1 t_2$$

$$Area\ D = t_2[2(Area\ E)/(T - t_2)]$$

$$Area\ E = [(T - t_2)(Area\ D/t_2)]/2$$

7.6 Optimization of the model

The idea is to optimize the total expected revenue by finding the values of the perfect quality product's price (s) and the expected cycle time (E[T]).

The objective function gives which values of (E[T]) and (s) maximizes the total profit function per each of the 5 variations of the mathematical models (one per each growth model).

The optimization process is done through a Hessian Matrix, the basic matrix for this optimization function is the following:

(37)

$$\textit{Hessian Matrix of } \mathrm{E}[\mathrm{TPU}] = \begin{bmatrix} L & M \\ M & N \end{bmatrix} = \begin{bmatrix} L = \dfrac{\partial^2 E[TPU]}{\partial (E[T])^2} & M = \dfrac{\partial^2 E[TPU]}{\partial E[T] \partial s} \\ M = \dfrac{\partial^2 E[TPU]}{\partial \mathrm{s} \partial \mathrm{E}[\mathrm{T}]} & N = \dfrac{\partial^2 E[TPU]}{\partial (\mathrm{s})^2} \end{bmatrix}$$

The optimization of the total expected revenue is obtained though this Hessian matrix. This is done using the Hessian theorem referenced as Theorem 9.6 in the book Vol. 2 Calculus and Linear Algebra, with Applications to Differential Equations and Probability (J. Wiley, 1969). In the original theorem in order to find the relative maximum all egeinvalues in H(a) must be negative. Since this is a quadratic hessian matrix, the second-derivative test for extrema of functions of two variables (J. Wiley, 1969) can be used interconvertibly as justified by Thorem 9.7 in the book quoted above. In this methodology, the former 9.7 theorem is favored for clarity reasons:

"Theorem 9.7 Let a be a stationary point of a scalar field $f(x_,, x_2)$ *with continous second-oder partial derivatives in a 2-ball* $B(u)$. *Let*

$$A = D_{1,1} f(a), \quad B = D_{1,2} f(a), \quad C = D_{2,2} f(a),$$

and let

$$A = \det H(a) = \det \begin{pmatrix} A & B \\ B & C \end{pmatrix} = AC - B^2$$

Then we have:

(...)
(c) If $A > 0$ and $A < 0, f$ has a relative maximum at a."

When applying the theorem 9.7 to the Hessian of the $E[TPU]$ formula, it can be concluded that in order to find the values of s and $E[T]$ that maximize the expected total revenue ($E[TPU]$), the following conditions must be true: $L < 0$, $N < 0$ and $LN - M^2 > 0$. As per reference to the original theorem 9.7: $L = A$, $B = N$ and $C = M$.

7.6.1 Optimization for each case (per growth function)

Since the distinction of each of the models is its growth function, the double derivative of E[T] and the cross derivatives of E[TPU] per s, an viceversa are the same per every value of w_t. N will be the only derivative that changes depending on the growth model. This means that the rest of derivatives will be the same for every model (excluding the respective growth functions) in the following functions:

(38) $$L = \frac{\partial^2 E[TPU]}{\partial (E[T])^2}$$ (39) $$M = \frac{\partial^2 E[TPU]}{\partial E[T] \partial s}$$ (40) $$M = \frac{\partial^2 E[TPU]}{\partial s \partial E[T]}$$

The cross derivatives (M) of the $E[TPU]$ function are the following:

(41) $$M = \frac{\partial^2 E[TPU]}{\partial s \partial E[T]} = h\varphi\beta s^{\varphi-1}\left[\frac{1}{2} + \frac{2(a - \beta s^{\varphi})\,E[x]}{\gamma(1 - E[x])^2}\right]$$

(42) $$M = \frac{\partial^2 E[TPU]}{\partial E[T] \partial s} = h\varphi b s^{\varphi-1}\left[\frac{1}{2} + \frac{2(a - \beta s^{\varphi})\,E[x]}{\gamma(1 - E[x])^2}\right]$$

The double derivative of E[TPU] with respect of $E[T]^2$ (L):

(43) $$L = \frac{\partial^2 E[TPU]}{\partial (E[T])^2} = \frac{-2K}{(E[T])^3}$$

Regarding the rest of the Hessian matrix, the double derivative of price (s), changes per each one of the five different growth functions (the 3 growth models taking into account that model III has 3 growth cases). Given the different w_t, the double derivative of E[TPU] with respect of price (s) the N derivatives, are the following per each model:

7.6.1.1 N for Model I (logistic growth function)

(44)

$$\frac{\partial E[TPU]}{\partial s^2} = -(\varphi b s^{\varphi-1}) \left[s + \frac{vE[x]}{1-E[x]} - \frac{pw_0}{w_1(1-E[x])} - \frac{z}{1-E[x]} - c\alpha \frac{\left[t_1 + \frac{1}{\lambda}\left[\ln(1+\beta e^{-\lambda t_1}) - \ln(1+\beta) \right] \right]}{w_1(1-E[x])} \right] - hE[T]\left(\frac{1}{2} + \frac{2E[x](a-bs^{\varphi})}{\gamma(1-E[x])^2} \right) + (a-bs^{\varphi}) = 0$$

7.6.1.2 N for Model II (linear growth function)

(45)

$$\frac{\partial E[TPU]}{\partial s^2} = -(\varphi b s^{\varphi-1}) \left[s + \frac{vE[x]}{1-E[x]} - \frac{pw_0}{w_1(1-E[x])} - \frac{z}{1-E[x]} - c \frac{\left[\frac{(w_1-w_0)^2}{2\gamma} \right]}{w_1(1-E[x])} \right] - hE[T]\left(\frac{1}{2} + \frac{2E[x](a-bs^{\varphi})}{\gamma(1-E[x])^2} \right) + (a-bs^{\varphi}) = 0$$

7.6.1.3 N for Model III case 1 (split linear growth function with target weight in the initial region)

(46)

$$\frac{\partial E[TPU]}{\partial s^2} = -(\varphi b s^{\varphi-1})$$

$$\left[s + \frac{vE[x]}{1-E[x]} - \frac{pw_0}{w_1(1-E[x])} - \frac{z}{1-E[x]} - c\frac{\left[\frac{(w_1-w_0)^2}{2\delta_1}\right]}{w_1(1-E[x])} \right]$$

$$-hE[T]\left(\frac{1}{2} + \frac{2E[x](a-bs^{\varphi})}{\gamma(1-E[x])^2}\right) + (a-bs^{\varphi}) = 0$$

7.6.1.4 N for Model III case 2 (split linear growth function with target weight in the intermediate growth region)

(47)

$$\frac{\partial E[TPU]}{\partial s^2} = -(\varphi b s^{\varphi-1})$$

$$\left[\begin{array}{l} s + \frac{vE[x]}{1-E[x]} - \frac{pw_0}{w_1(1-E[x])} - \frac{z}{1-E[x]} \\ -c\frac{\left[\frac{(w_1-w_0)^2}{2\delta_1} + \frac{(w_1-w'_1)^2}{2\delta_2} + \frac{(w_1-w'_1)(w'_1-w_0)}{\delta_2}\right]}{w_1(1-E[x])} \end{array} \right]$$

$$-hE[T]\left(\frac{1}{2} + \frac{2E[x](a-bs^{\varphi})}{\gamma(1-E[x])^2}\right) + (a-bs^{\varphi}) = 0$$

7.6.1.5 N for Model III case 3 (split linear growth function with target weight in the last growth region)

(48)

$$\frac{\partial E[TPU]}{\partial s^2} = -(\varphi b s^{\varphi-1})$$

$$\left[s + \frac{vE[x]}{1-E[x]} - \frac{pw_0}{w_1(1-E[x])} - \frac{z}{1-E[x]} -c\frac{\left[\frac{(w'_1-w_0)^2}{2\delta_1} + \frac{(w''_1-w'_1)^2}{2\delta_2} + (t''_1-t'_1)(w'_1-w_0) + \frac{(w_1-w''_1)^2}{2\delta_3} + \frac{(w_1-w''_1)(w''_1-w_0)}{\delta_3}\right]}{w_1(1-E[x])} \right]$$

$$-hE[T]\left(\frac{1}{2} + \frac{2E[x](a-bs^{\varphi})}{\gamma(1-E[x])^2}\right) + (a-bs^{\varphi}) = 0$$

8. Model testing

Model testing consists on programming the model in the optimization software Lingo. In this phase, there are three main steps:

1) Checking that the optimization and mathematical model do not have any error by testing with the data from the original model and seeing if same results are obtained.
2) Adding the price optimization part in order to compare the results of this model to the ones in the original paper, aiming to see if it does improve the results.
3) Adding the emission costs part to the model in the software.

8.1 Checking optimization and mathematical correctness

Ensuring that the optimization and mathematical model are correct has been done by utilizing the data from the original paper (Makoena Sebatjane & O. Adetunji, 2019a). All models are programmed on Lingo without the additions done in this research, then the data from the original paper is plugged in the Hessian optimization matrix, since the same results as those in the original paper (table 4) were obtained, the model and the mathematical formulation are correct.

The data used for the calibration of the model was obtained from the numerical example in point 5.1 of the original paper “Economic order quantity model for growing items with imperfect quality” by Makoena Sebatjane & O. Adetunji (2019). All five models (3 models with the third one having 3 cases) will use the parameters given in this numerical example.

8.2 Price optimization test

After checking that the mathematical model was correct, the price optimization part was done. To the 5 models, the price optimization formulation was added, keeping all other data identical to the original model as to test what is the improvement from the original model. The results of the model with price optimization appear compared to the original paper in table 4 to see if it is able to achieve a better result when price (s) is an optimization variable. Screenshots of the program results and further information on the iterations can be found in the annex. The 5 different Lingo optimization programs (one per each growth case) can be found attached as an annex as well.

Table 4. Summary of the testing results obtained from the program

Model	**E[tpu] Original paper** [Sebatjane & Adentuji (2019)]	**E[tpu] This research project** [With price optimization]	**Improvement** [in percentage]
Model I	34.641,73	44.038,89	27,13%
Model II	30.964.01	41.532,12	34,13%
Model III Case 1	33.746,67	38.480,89	14,03%
Model III Case 2	34.015,80	43.607,37	28,20%
Model III Case 3	142194.7	147689.5	3.86%

Source: Own elaboration. Numerical values from obtained from the Lingo programs attached in the anex, original data is from section 5. "Numerical results" in Sebatjane & Adentuji's 2019 paper on an EOQ model for growing items with imperfect quality.

8.3 Emission costs in the program

Last step is to add the emission costs. In this step, it is known that the mathematical formulation is correct and the use of price optimization improves the results of the model in all the 5 cases as Table 4 shows.

The addition of emission costs cannot be compared to the original paper, as it is a particular addition to this project and the data provided in Sebatjane and Adentuji's 2019 does not provide data on these variables. Regardless, this research project provides the final model (attached in the annex) with both price optimization and emission costs coded in Lingo.

The reader is able to use the programs for all 5 cases. Variables are already in the program and its naming coincides with those explained in section 7.2. For the emission cost programs, the reader will only need to enter the values for the variables used for the emission costs' part and the program will find the optimal cycle time ($E[t]$) and price(s) .

9. Results

Table 4 illustrates the results of each program when price becomes a decision variable in the optimization process using the data from the original paper by Sebatjane and Adentuji (2019). Model 1 with items that follow a logistic growth function has an improvement of 27,13% going from 34.641,73 to 44.038,89 in expected total revenue. Model 2 using a linear growth function improves from 30.964.01 in the original paper to 41.532,12, a 34,13% when price optimization is introduced in the model. The last model, model 3 has three use cases, the first one, where linear growth is allocated in the first region, the improvement is a 14,03% moving from 33.746,67 to 38.480,89 in expected total profit. For the second and third cases the results improve by 28,20% and 3,86% respectively.

10. Conclusion

The main optimization model (table 4) shows improvement in all cases when incorporating growing items and price optimization. Environmental costs cannot be compared to former data results as there are none but this feature offers more granularity and adaptability to the EOQ model, offering the reader the choice to add it or not as chapter 7.4.3.6. shows through the formulation.

The model has been adapted successfully to 5 different growth models which portray most of the growth patterns used in EOQ growing items literature. In all cases efficiency was improved when more granularity to demand was introduced. Case 3 of model III stands out because it moves around the 140.000 E[tpu], when the rest of cases oscillate around the 30.000 E[tpu] and it offers the least improvement by far (3,86%). This difference is probably due to the variables not being independent in this case but to confirm it, further investigation would be required. This conclusion is particularly interesting because it brings up new opportunities for research, regarding variable behavior and opportunity for price optimization in split linear growth functions with target weight at the end of the cycle (Model III case 3) versus other growth functions in EOQ models.

From the results in table 4, it can be stated that those items with linear growth patterns (Model II) benefit the most when introducing price optimization and demand granularity. Nonetheless, in future research it would be valuable to test more data sets against different growth patterns so this results can be further contrasted. It would be advisable to use data similar to the one included in this project but with CO2 emission costs for items with different growth patterns, since this would provide more insights on the model and its effectiveness at maximizing profits with CO2 constraints.

On the continuity of this research, it offers the opportunity to further explore the impact of demand function adequacy related to item growth patterns when optimizing an EOQ. As secondary branches, the research of CO2 emissions linked to demand is interesting at a business level because it offers more granularity of demand while incorporating the tendency to consider sustainability a part of the customer's choice not just a cost of production. This sustainability in marketing could be incorporated as a variable of the elasticity of demand. While marketing is often considered in EOQ models, proof being the literature on price optimization models and demand functions, marketing oriented to sustainability of the supply chain and inventory policies is not often included. This linked to the increasing social conscience towards environmental impact offers a great opportunity to explore this line in EOQ research.

Overall, this project opens a new line of research in the Logistics and Maritime Business Bachelor in Tecnocampus. This branch moves towards decision making in inventory optimization, which considers business interests, marketing and reduction of negative externalities by incorporating environmental impact. The reduction of negative externalities in the model is particularly interesting because it is able to translate an economic concept that intends to represent social impact and business trends into a quantifiable term applicable in business decision making. These last characteristics are especially relevant in the context of the opening of a new line of research for the Bachelor, as they offer future students the opportunity to further develop this research by including their own experiences in the field and their background in classic economic theory.

10. Bibliography

Absi, N., Dauzère-Pérès, S., Kedad-Sidhoum, S., Penz, B., Rapine, C. (2013). *Lot sizing with carbon emission constraints. European Journal of Operational Research*. *227(1)*, 55-61.

Alamri, A. A., Harris, I., & Syntetos, A. A. (2016). Efficient inventory control for imperfect quality items. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2016.03.058

Arcelus, F. J., & Srinivasan, G. (1987). Inventory policies under various optimizing criteria and variable markup rates. *Management Science*, *33*(6), 756–762. https://doi.org/10.1287/mnsc.33.6.756

Arslan, M. C., Turkay, M. (2013). (2013). EOQ revisited with sustainability considerations. *Foundations of Computing and Decision Sciences, 38(4)*, 223-249.

Battini, D., Persona, A., Sgarbossa, F. (2014). *A sustainable EOQ model: Theoretical formulation and applications. Internatio*(149), 145–153.

Benjaafar, S., Li, Y., Daskin, M. (2012). Carbon footprint and the management of supply chains: Insights from simple models. *IEEE Transactions on Automation Science and Engineering*, *10(1)*, 99–116.

Bonney, M., Jaber, M. Y. (2011). Environmentally responsible inventory models: Non-classical models for a non-classical era. *International Journal of Production Economics*, *133(1)*, 43–53.

Bozorgi, A., Pazour, J., Nazzal, D. (2014). (2014). A new inventory model for cold items that considers costs and emissions. *International Journal of Production Economics, 155*, 114–125.

Bozorgi, A. (2016). Multi-product inventory model for cold items with cost and emission consideration. *International Journal of Production Economics, 176*, 123–142.

Cárdenas-Barrón, L. E. (2012). A complement to “A comprehensive note on: An economic order quantity with imperfect quality and quantity discounts.” *Applied Mathematical Modelling*, *36*(12), 6338–6340. https://doi.org/10.1016/j.apm.2012.02.021

Chan, W. M., Ibrahim, R. N., Lochert, P. B. (2003). A new EPQ model: integrating lower pricing, rework and reject situations. *Production Planning and Control*, *14*(7), 588–595.

Chang, H. C. (2011). A comprehensive note on: an economic order quantity with imperfect quality and quantity discounts. 5208-5216. *Applied Mathematical Modelling*, *35*(10), 5208–5216.

Chang, Hung Chi. (2004). An application of fuzzy sets theory to the EOQ model with imperfect quality items. *Computers and Operations Research*. https://doi.org/10.1016/S0305-0548(03)00166-7

Chang, Hung Chi, & Ho, C. H. (2010). Exact closed-form solutions for "optimal inventory model for items with imperfect quality and shortage backordering." *Omega*, *38*(3–4), 233–237. https://doi.org/10.1016/j.omega.2009.09.006

Chen, X., Benjaafar, S., Elomri, A. (2013). The carbon-constrained EOQ. *Operations Research Letters*, *41(2),* 172-179.

Chen, L. H., & Kang, F. Sen. (2010). Coordination between vendor and buyer considering trade credit and items of imperfect quality. *International Journal of Production Economics*, *123*(1), 52–61. https://doi.org/10.1016/j.ijpe.2009.06.043

Chen, Y., Ray, S., & Song, Y. (2006). Optimal pricing and inventory control policy in periodic-review systems with fixed ordering cost and lost sales. *Naval Research Logistics*, *53*(2), 117–136. https://doi.org/10.1002/nav.20127

Chung, K. J., Her, C. C., & Lin, S. Der. (2009). A two-warehouse inventory model with imperfect quality production processes. *Computers and Industrial Engineering*. https://doi.org/10.1016/j.cie.2008.05.005

Climate Change – United Nations Sustainable Development. (n.d.). Retrieved December 27, 2020, from https://www.un.org/sustainabledevelopment/climate-change/

Corporate Honesty and Climate Change: Time to Own Up and Act | NRDC. (n.d.). Retrieved December 28, 2020, from https://www.nrdc.org/experts/josh-axelrod/corporate-honesty-and-climate-change-time-own-and-act

Eliashberg, J., & Steinberg, R. (1993). Marketing-production joint decision-making. In *Handbooks in Operations Research and Management Science* (Vol. 5, Issue C). https://doi.org/10.1016/S0927-0507(05)80041-6

Erlenkotter, D. (1990). Ford Whitman Harris and the Economic Order Quantity Model. *Operations Research*, *38*(6), 937–946. https://doi.org/10.1287/opre.38.6.937

Forests, desertification and biodiversity – United Nations Sustainable Development. (n.d.). Retrieved December 27, 2020, from https://www.un.org/sustainabledevelopment/biodiversity/

Gharaei, A., & Almehdawe, E. (2019). Economic growing quantity. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2019.107517

Goyal, S. K., & Cárdenas-Barrón, L. E. (2002). Note on: Economic production quantity model for items with imperfect quality - A practical approach. *International Journal of Production Economics*. https://doi.org/10.1016/S0925-5273(01)00203-1

Harris, F.W. (1913). art12. *Factory the Magazine of Management*.

Harris, Ford W. (1913). How many parts to make at once. In *The Magazine of Management* (Vol. 152, Issue 2, pp. 135–136). https://doi.org/10.1016/j.ijpe.2014.07.003

Hsu, W. K., & Yu, H. F. (2011). An EOQ model with imperfective quality items under an announced price increase. *Journal of the Chinese Institute of Industrial Engineers*, *28*(1), 34–44.

Hsu, W. K. K., Yu, H. F. (2009). EOQ model for imperfective items under a one-time-only discount. *Omega*, *37(5)*, 1018–1026.

Hua, G., Cheng, T. C. E., Wang, S. (2011). Managing carbon footprints in inventory management. *International Journal of Production Economics*, *132(2)*, 178-185.

Huang, Jian, Leng, M., & Parlar, M. (2013). Demand functions in decision modeling: A comprehensive survey and research directions. *Decision Sciences*, *44*(3), 557–609. https://doi.org/10.1111/deci.12021

Huang, Junming, Gates, A. J., Sinatra, R., & Barabási, A. L. (2020). Historical comparison of gender inequality in scientific careers across countries and disciplines. *Proceedings of the National Academy of Sciences of the United States of America*, *117*(9), 4609–4616. https://doi.org/10.1073/pnas.1914221117

Jaber, M. Y., Glock, C. H., El Saadany, A. M. (2013). Supply chain coordination with emissions reduction incentives. *International Journal of Production Research, 51(1)*, 69-82.

Jaber, M. Y., Goyal, S. K., & Imran, M. (2008). Economic production quantity model for items with imperfect quality subject to learning effects. *International Journal of Production Economics*, *115*(1), 143–150. https://doi.org/10.1016/j.ijpe.2008.05.007

Jaber, Mohamad Y., Zanoni, S., & Zavanella, L. E. (2013). An entropic economic order quantity (EnEOQ) for items with imperfect quality. *Applied Mathematical Modelling*. https://doi.org/10.1016/j.apm.2012.07.046

Kazemi, N., Abdul-Rashid, S. H., Ghazilla, R. A. R., Shekarian, E., & Zanoni, S. (2018). Economic order quantity models for items with imperfect quality and emission considerations. *International Journal of Systems Science: Operations and Logistics*. https://doi.org/10.1080/23302674.2016.1240254

Khalilpourazari, S., & Pasandideh, S. H. R. (2019). Modeling and optimization of multi-item multi-constrained EOQ model for growing items. *Knowledge-Based Systems*. https://doi.org/10.1016/j.knosys.2018.10.032

Khan, M., Jaber, M. Y., Guiffrida, A. L., & Zolfaghari, S. (2011). A review of the extensions of a modified EOQ model for imperfect quality items. In *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2011.03.009

Khan, M., Jaber, M. Y., & Wahab, M. I. M. (2010). Economic order quantity model for items with imperfect quality with learning in inspection. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2009.10.011

Khan, Mehmood, Jaber, M. Y., & Bonney, M. (2011). An economic order quantity (EOQ) for items with imperfect quality and inspection errors. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2010.01.023

Khan, Mehmood, Jaber, M. Y., Zanoni, S., & Zavanella, L. (2016). Vendor managed inventory with consignment stock agreement for a supply chain with defective items. *Applied Mathematical Modelling*, *40*(15–16), 7102–7114. https://doi.org/10.1016/j.apm.2016.02.035

Lin, T. Y., Sarker, B. R. (2017). A pull system inventory model with carbon tax policies and imperfect quality items. *Applied Mathematical Modelling*, *50*, 450–462.

Lin, T. Y. (2010). An economic order quantity with imperfect quality and quantity discounts. *Applied Mathematical Modelling*, *34*(10), 3158–3165.

Maddah, B., & Jaber, M. Y. (2008a). Economic order quantity for items with imperfect quality: Revisited. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2007.07.003

Maddah, B., & Jaber, M. Y. (2008b). Economic order quantity for items with imperfect quality: Revisited. *International Journal of Production Economics*, *112*(2), 808–815. https://doi.org/10.1016/j.ijpe.2007.07.003

Malekitabar, M., Yaghoubi, S., & Gholamian, M. R. (2019). A novel mathematical inventory model for growing-mortal items (case study: Rainbow trout). *Applied Mathematical Modelling*, *71*, 96–117.

Mapping the benefits of a circular economy. (n.d.). Retrieved December 27, 2020, from https://www.mckinsey.com/business-functions/sustainability/our-insights/mapping-the-benefits-of-a-circular-economy

Mestrallet, G. (2015, December 11). *Why carbon pricing is key to our renewable future | World Economic Forum*. World Economic Forum. https://www.weforum.org/agenda/2015/12/why-carbon-pricing-is-key-to-our-renewable-future

Modak, N. M., Panda, S., & Sana, S. S. (2015). Optimal just-in-time buffer inventory for preventive maintenance with imperfect quality items. *Tékhne*. https://doi.org/10.1016/j.tekhne.2016.02.002

New Circular Economy Strategy - Environment - European Commission. (n.d.). Retrieved December 27, 2020, from https://ec.europa.eu/environment/circular-economy/

Nobil, A. H., Sedigh, A. H. A., & Cárdenas-Barrón, L. E. (2019). A Generalized Economic Order Quantity Inventory Model with Shortage: Case Study of a Poultry Farmer. *Arabian Journal for Science and Engineering*, *44*(3), 2653–2663. https://doi.org/10.1007/s13369-018-3322-z

Organización de las Naciones Unidas para la Alimentación y la Agricultura. (2006). *FAO - Noticias: Key facts and findings*. http://www.fao.org/news/story/es/item/197623/icode/

Oum, T. H. (1989). Alternative demand models and their elasticity estimates. *Journal of Transport Economics and Policy*, *23*(2), 163–187.

Papachristos, S., & Konstantaras, I. (2006). Economic ordering quantity models for items with imperfect quality. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2004.11.004

Paul, S., Wahab, M. I. M., Ongkunaruk, P. (2014). Joint replenishment with imperfect items and price discount. *Computers & Industrial Engineering*, *74*, 179–185.

Proust, M. (1923). *A la recherche du temps perdu, La Prisonnière*. Indepent.

Rad, M. A., Khoshalhan, F., Glock, C. H. (2018). Optimal production and distribution policies for a two-stage supply chain with imperfect items and price-and advertisement-sensitive demand: A note. *Applied Mathematical Modelling*, *57*, 625–632.

Rezaei, J. (2014). Economic order quantity for growing items. *International Journal of Production Economics*, *155*, 109–113. https://doi.org/10.1016/j.ijpe.2013.11.026

Rezaei, J. (2016). Economic order quantity and sampling inspection plans for imperfect items. *Computers and Industrial Engineering*, *96*, 1–7. https://doi.org/10.1016/j.cie.2016.03.015

Rezaei, J., & Salimi, N. (2012). Economic order quantity and purchasing price for items with imperfect quality when inspection shifts from buyer to supplier. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2012.01.005

Rosič, H., Jammernegg, W. (2013). The economic and environmental performance of dual sourcing: A newsvendor approach. *International Journal of Production Economics*, *143(1)*, 109–119.

Sadjadi, S. J., Yazdian, S. A., & Shahanaghi, K. (2012). Optimal pricing, lot-sizing and marketing planning in a capacitated and imperfect production system. *Computers and Industrial Engineering*, *62*(1), 349–358. https://doi.org/10.1016/j.cie.2011.10.006

Salameh, M. K., & Jaber, M. Y. (2000). Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*. https://doi.org/10.1016/S0925-5273(99)00044-4

Sebatjane, M. (2019). *Selected deterministic models for lot sizing of growing items inventory*. University of Pretoria.

Sebatjane, Makoena, & Adetunji, O. (2019a). Economic order quantity model for growing items with imperfect quality. *Operations Research Perspectives*, *6*. https://doi.org/10.1016/j.orp.2018.11.004

Sebatjane, Makoena, & Adetunji, O. (2019b). Economic order quantity model for growing items with incremental quantity discounts. *Journal of Industrial Engineering International*, *15*(4), 545–556. https://doi.org/10.1007/s40092-019-0311-0

Shekarian, E., Olugu, E. U., Abdul-Rashid, S. H., Kazemi, N. (2016). An economic order quantity model considering different holding costs for imperfect quality items subject to fuzziness and learning. *Journal of Intelligent & Fuzzy Systems*, *30*(5), 2985–2997.

State and Trends of Carbon Pricing 2020. (2020). https://doi.org/10.1596/978-1-4648-1586-7

Sustainable consumption and production – United Nations Sustainable Development. (n.d.). Retrieved December 27, 2020, from https://www.un.org/sustainabledevelopment/sustainable-consumption-production/

Taleizadeh, A. A., Khanbaglo, M. P. S., & Cárdenas-Barrón, L. E. (2016). An EOQ inventory model with partial backordering and reparation of imperfect products. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2016.09.013

The Sustainable Development Agenda – United Nations Sustainable Development. (n.d.). Retrieved December 27, 2020, from https://www.un.org/sustainabledevelopment/development-agenda/

Tiwari, S., Daryanto, Y., & Wee, H. M. (2018). Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. *Journal of Cleaner Production*, *192*, 281–292. https://doi.org/10.1016/j.jclepro.2018.04.261

Wahab, M. I. M., & Jaber, M. Y. (2010). Economic order quantity model for items with imperfect quality, different holding costs, and learning effects: A note. *Computers and Industrial Engineering*. https://doi.org/10.1016/j.cie.2009.07.007

Wahab, M. I. M., Mamun, S. M. H., & Ongkunaruk, P. (2011). EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact. *International Journal of Production Economics*. https://doi.org/10.1016/j.ijpe.2011.06.008

Wang, W. T., Wee, H. M., Cheng, Y. L., Wen, C. L., & Cárdenas-Barrón, L. E. (2015). EOQ model for imperfect quality items with partial backorders and screening constraint. *European Journal of Industrial Engineering*, *9*(6), 744–773. https://doi.org/10.1504/EJIE.2015.074384

Wee, H. M., Yu, J., & Chen, M. C. (2007). Optimal inventory model for items with imperfect quality and shortage backordering. *Omega*, *35*(1), 7–11. https://doi.org/10.1016/j.omega.2005.01.019

Whitin, T. M. (1955). Inventory Control and Price Theory. *Management Science*, *2*(1), 61–68. https://doi.org/10.1287/mnsc.2.1.61

Wiley, J. (1969). *CALCULUS Vol. 2 Multi Variable Calculus and Linear Algebra, with Applications to DifFeren tial Equations and Probability SECOND EDITION*.

World Economic Forum., & Bartlett, N. (2016). *Seven key trends in corporate carbon pricing | World Economic Forum*. World Economic Forum. https://www.weforum.org/agenda/2016/11/seven-key-trends-in-corporate-carbon-pricing

Yoo, S. H., Kim, D., Park, M. S. (2009). Economic production quantity model with imperfect-quality items, two-way imperfect inspection and sales return. *International Journal of Production Economics*, *121*(1), 255–265.

Yu, J. C. P., Wee, H. M., & Chen, J. M. (2005). Optimal ordering policy for a deteriorating item with imperfect quality and partial backordering. *Journal of the Chinese Institute of Industrial Engineers*, *22*(6), 509–520. https://doi.org/10.1080/10170660509509319

Zhang, Y., Li, L. Y., Tian, X. Q., & Feng, C. (2016). Inventory management research for growing items with carbon-constrained. *Chinese Control Conference, CCC*, *2016-August*, 9588–9593. https://doi.org/10.1109/ChiCC.2016.7554880

Zhou, Y. W., Chen, J., Wu, Y., Zhou, W. (2015). EPQ models for items with imperfect quality and one-time-only discount. *Applied Mathematical Modelling*, *39*(3–4), 1000-1018.